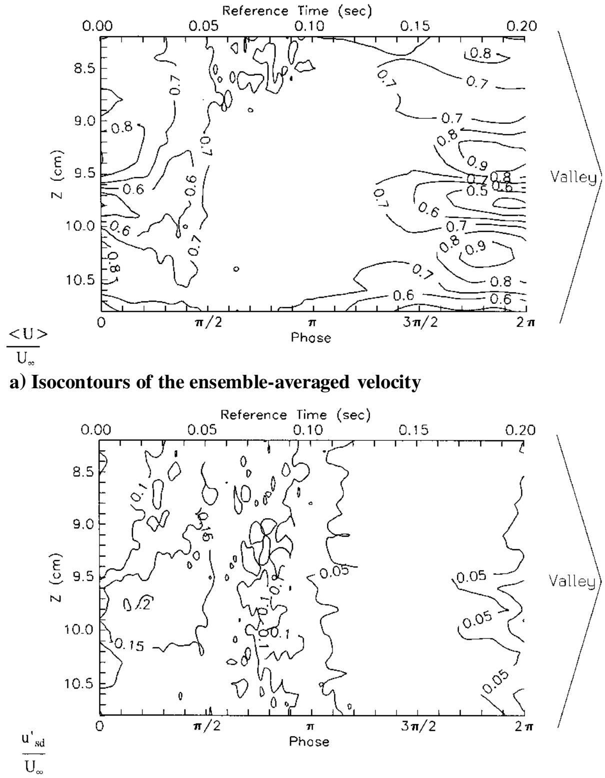

a) Isocontours of the ensemble-averaged velocity

b) Standard deviation

Fig. 3 Quantitative results of the interaction taken at x = 82.5 cm, y = 0.3 cm (y^+ = 34): Saw-tooth valley is located at z = 9.5 cm.

centered about the low-speed streak ($z \approx 9.8$ cm). Because the high-speed wallward (insweep) motion occurs at $\phi \approx 0$, the wall eddy breaks down soon after the arrival of the insweep at that location.

Breakdown phase angles were quite different for wall eddies located near the sawtooth tip. In this case, the streak structure was terminated during $0.4\pi \lesssim \phi \lesssim \pi$, which is shorter in duration than the valley region. This difference in breakdown phase angle was due to the late arrival of the high-speed insweep at the tip location. Thus, breakdown of wall eddy is driven by the arrival of the high-speed insweep, which is associated with the three-dimensional, outer-region structure.

Summary

An experiment to study the interactive effect of large, three-dimensional outer eddies on small streamwise vortices was conducted. The TBL near-wall structure was emulated by Görtler streamwise vortices. Outer-region, three-dimensional eddies were shed from the pitch oscillation of an airfoil with sawtooth trailing edge. The three-dimensional eddies included skewed leading and trailing edges and a wallward motion similar to insweeps observed in TBLs. The interaction involved fluctuation of the near-wall structure as the outer eddy approached. Violent mixing of fluid and breakdown of the near-wall structure ensued as high-speed fluid associated with the outer eddy arrived. This suggests that the insweep associated with the outer region helps trigger the bursting of the near-wall structure in bounded turbulent shear flows.

Acknowledgments

This work was sponsored by the Office of Naval Research under Contract N00014-89-J-1400 and the U.S. Air Force Office of Scientific Research under Contract F49620-85-C-0080.

References

[1]Cantwell, B. J., "Organized Motion in Turbulent Flow," *Annual Review of Fluid Mechanics*, Vol. 13, 1981, pp. 457–515.

[2]Robinson, S. K., "Coherent Motions in the Turbulent Boundary Layer," *Annual Review of Fluid Mechanics*, Vol. 23, 1991, pp. 601–639.

[3]Kline, S. J., Reynolds, W. C., Schraub, F. A., and Runstadler, P. W., "The Structure of Turbulent Boundary Layers," *Journal of Fluid Mechanics*, Vol. 30, Dec. 1967, pp. 741–773.

[4]Bogard, D. G., and Tiederman, W. G., "Characteristics of Ejections in Turbulent Channel Flow," *Journal of Fluid Mechanics*, Vol. 179, June 1987, pp. 1–19.

[5]Acarlar, M. S., and Smith, C. R., "A Study of Hairpin Vortices in a Laminar Boundary Layer. Part 1. Hairpin Vortices Generated by a Hemisphere Protuberance," *Journal of Fluid Mechanics*, Vol. 175, Feb. 1987, pp. 1–41.

[6]Swearingen, J. D., and Blackwelder, R. F., "The Growth and Breakdown of Streamwise Vortices in the Presence of a Wall," *Journal of Fluid Mechanics*, Vol. 182, Sept. 1987, pp. 255–290.

[7]Blackwelder, R. F., and Kovasznay, L. S., "Time Scales and Correlations in a Turbulent Boundary Layer," *Physics of Fluids*, Vol. 15, Sept. 1972, pp. 1545–1554.

[8]Brown, G. L., and Thomas, A. S. W., "Large Structure in a Turbulent Boundary Layer," *Physics of Fluids*, Vol. 20, Oct. 1977, pp. S243–S252.

[9]Chen, C. P., and Blackwelder, R. F., "Large-Scale Motion in a Turbulent Boundary Layer: A Study Using Temperature Contamination," *Journal of Fluid Mechanics*, Vol. 89, Nov. 1978, pp. 1–31.

[10]Praturi, A. K., and Brodkey, R. S., "A Stereoscopic Visual Study of Coherent Structures in Turbulent Shear Flow," *Journal of Fluid Mechanics*, Vol. 89, Nov. 1978, pp. 251–272.

[11]Smith, C. R., "Visualization of Turbulent Boundary-Layer Structure Using a Moving Hydrogen Bubble-Wire Probe," *Coherent Structures of Turbulent Boundary Layers*, edited by C. R. Smith and D. E. Abbott, Lehigh Univ. Press, Bethlehem, PA, 1978, pp. 48–97.

[12]Falco, R., "The Production of Turbulence Near a Wall," AIAA Paper 80-1356, July 1980.

[13]Myose, R. Y., and Blackwelder, R. F., "On the Role of the Outer Region in the Turbulent-Boundary-Layer Bursting Process," *Journal of Fluid Mechanics*, Vol. 259, Jan. 1994, pp. 345–373.

[14]Myose, R. Y., and Iwata, J., "Flow Visualization of an Oscillating Airfoil with Saw-Tooth Trailing Edge," *AIAA Journal*, Vol. 34, No. 9, 1996, pp. 1748–1750.

F. W. Chambers
Associate Editor

Constrained Damage Detection Technique for Simultaneously Updating Mass and Stiffness Matrices

Jason Kiddy* and Darryll Pines†
University of Maryland,
College Park, Maryland 20742-3015

Introduction

OVER the past 10 years, a considerable amount of attention has been paid to modal-based damage detection algorithms. In most instances, the damage detection problem is simplified to the detection and characterization of stiffness failures. However, several authors have attempted to simultaneously classify both mass and stiffness changes.[1,2] Recently, it has been noted that modal data cannot be used to correctly identify these combined failures due to the nonunique nature of the mode shapes.[3] This arbitrary scaling of the mode shapes creates an infinite number of mass and stiffness matrices that satisfy the eigenvalue equation. However, it will be shown that it is possible to perform simultaneous updating of both matrices if the problem is properly constrained.

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*Graduate Fellow, Department of Aerospace Engineering. Student Member AIAA.

†Assistant Professor, Department of Aerospace Engineering. Senior Member AIAA.

Analytical Development

The standard eigenvalue equation is given as

$$[K][\phi] = [\lambda][M][\phi] \tag{1}$$

where $[\lambda]$ and $[\phi]$ are the $n \times n$ eigenvalue and eigenvector matrices, respectively, for the $n \times n$ system mass and stiffness matrices. The modal matrices can be normalized such that

$$\begin{aligned} [\phi]^t[K][\phi] &= [\lambda] \\ [\phi]^t[M][\phi] &= [I] \end{aligned} \tag{2}$$

where $[I]$ is the identity matrix and $[\ ]^t$ denotes the transpose of a matrix.

In a practical situation, the experimentally measured mode shapes are arbitrarily scaled. These experimental, unscaled mode shapes $[\Phi]$ can be related to the normalized mode shapes $[\phi]$ by

$$[\Phi] = [d][\phi] \tag{3}$$

where $[d]$ is a diagonal matrix of scaling values. Placing these new arbitrary mode shapes into Eq. (2) gives

$$\begin{aligned} [\Phi]^t[K][\Phi] &= [d^2\lambda] \\ [\Phi]^t[M][\Phi] &= [d^2][I] \end{aligned} \tag{4}$$

It is apparent that the original mass and stiffness matrices do not normalize the measured mode shapes. However, there exist mass and stiffness matrices ($[\bar{M}]$ and $[\bar{K}]$, respectively) that satisfy the eigenvalue equation (1) and can be used to normalize the scaled mode shapes:

$$\begin{aligned} [\Phi]^t[\bar{K}][\Phi] &= [\lambda] \\ [\Phi]^t[\bar{M}][\Phi] &= [I] \end{aligned} \tag{5}$$

Therefore, there is an infinite number of mass and stiffness matrices that satisfy the eigenvalue equation. These matrices are dependent on the mode shape scaling factors $[d]$. Baruch[3] points out that this makes it impossible to uniquely update both the mass and stiffness matrices simultaneously.

However, this analysis can be extended to show that a method of updating both matrices simultaneously exists if a single constraint is applied. By solving the normalization equations (2) and (5) for the mass and stiffness matrices and comparing the two solutions, the following relationships can be developed:

$$\begin{aligned} [\bar{K}] &= [d^2]^{-1}[K] \\ [\bar{M}] &= [d^2]^{-1}[M] \end{aligned} \tag{6}$$

This shows that, although there is an infinite number of matrices that will satisfy the eigenvalue equation, all of these matrices are scaled versions of the same matrix. This statement can be validated by starting with Eq. (5) and making the appropriate substitutions to arrive at Eq. (2):

$$[\Phi]^t[\bar{M}][\Phi] = [d][\phi]^t[d^2]^{-1}[M][d][\phi] = [\phi]^t[M][\phi] \tag{7}$$

Herein lies the ability to simultaneously detect changes in both mass and stiffness. If a single element in either the mass or stiffness matrix is held constant, the updated matrices will be scaled around this element. To demonstrate this, represent the damaged matrices $[K]^d$ and $[M]^d$ by the following expansions:

$$[K]^d = \sum_{i=1}^{N} a_i K_i = a_1 K_1 + a_2 K_2 + \cdots + a_N K_N \tag{8}$$

$$[M]^d = \sum_{i=1}^{N} b_i M_i = b_1 M_1 + b_2 M_2 + \cdots + b_N M_N \tag{9}$$

where N is the number of structural elements, the scalars a_i and b_i represent the amount of change in a given element, and M_i and K_i represent the elemental mass and stiffness matrices.

Now constrain an element ($i = k$) to be undamaged. This implies that $a_k = b_k = 1$ and results in the following expressions:

$$[\bar{K}]^d = \sum_{i=1\ i\neq k}^{N} a_i K_i + K_k \tag{10}$$

$$[\bar{M}]^d = \sum_{i=1\ i\neq k}^{N} b_i M_i + M_k \tag{11}$$

But recall that there exists an infinite number of scaled damaged matrices that satisfy the damaged eigenvalue equation. These matrices have the following form:

$$\begin{aligned} [\bar{K}]^d &= [d^2]^{-1}[K]^d \\ [\bar{M}]^d &= [d^2]^{-1}[M]^d \end{aligned} \tag{12}$$

Substituting for the scaled damaged mass and stiffness matrices into Eq. (12) leads to the following set of N system equations with N unknowns:

$$[d^2]^{-1}[K]^d - \sum_{i=1\ i\neq k}^{N} a_i K_i = K_k \tag{13}$$

$$[d^2]^{-1}[M]^d - \sum_{i=1\ i\neq k}^{N} b_i M_i = M_k \tag{14}$$

The preceding set of system equations permits the unique determination of the coefficients a_i and b_i along with the scaling factor d. Hence, solutions for these scalars can be found by constraining any element in the structural model.

If no element is damaged (or incorrectly modeled), the resulting matrices for the updated system will be analogous to the original model. If, by chance, the constrained element is damaged ($a_k, b_k \neq 1$), every updated element will be rescaled. This will result in the updated mass and stiffness matrices being scaled differently from those of the original model. If this were to occur, the engineer would be faced with two possibilities. Either it would indicate uniform damage in every nonconstrained element, or it would indicate that the constrained element is the damaged element.

An alternate way of constraining the updating problem is to limit the maximum amount of change possible in the elements. This constraint may be posed such that the stiffness can only be increased or decreased by 95% in an element ($a_i > 0.05$). After updating, the new matrices can then be rescaled to correspond to the original system. This is done by choosing an element that does not show any damage, e.g., an element that is reduced the same amount as all adjacent elements, and by rescaling the updated matrices to this element ($a_k = 1$). Similar constraints can be placed on the mass matrix. If the mass of every element is constrained to be undamaged ($b_i = 1$), this is analogous to the problem that most researchers consider, where only stiffness changes are allowed.

Example of Updating Problem

The following example will demonstrate the preceding approach. Consider a simple cantilevered beam under axial loading as seen in Fig. 1. For simplicity, let each element be of unit length. The corresponding mass and stiffness matrices are

$$[M] = \frac{1}{6}\begin{bmatrix} 2m_1 + 2m_2 & m_2 & 0 \\ m_2 & 2m_2 + 2m_3 & m_3 \\ 0 & m_3 & 2m_3 \end{bmatrix}$$

$$[K] = \begin{bmatrix} EA_1 + EA_2 & -EA_2 & 0 \\ -EA_2 & EA_2 + EA_3 & -EA_3 \\ 0 & -EA_3 & EA_3 \end{bmatrix} \tag{15}$$

For the initial (undamaged) case, assume that all three elements are identical with $m_1 = m_2 = m_3 = 1$ and $EA_1 = EA_2 = EA_3 = 1$.

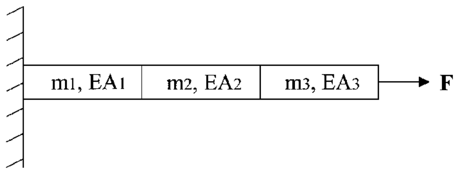

Fig. 1 Finite element model.

The undamaged matrices and corresponding eigenvalues and eigenvectors for this system are

$$[M] = \frac{1}{6}\begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \qquad [K] = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \tag{16}$$

$$[\lambda] = \begin{bmatrix} 9.8734 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0.2805 \end{bmatrix}$$

$$[\Phi] = \begin{bmatrix} 0.3536 & 0.7071 & 0.3536 \\ -0.6124 & 0 & 0.6124 \\ 0.7071 & -0.7071 & 0.7071 \end{bmatrix} \tag{17}$$

A simulated damage case can be considered where the mass and stiffness of the second element are reduced by 50 and 75%, respectively. The modal matrices for the damaged system are

$$[\lambda] = \begin{bmatrix} 9.3093 & 0 & 0 \\ 0 & 2.5327 & 0 \\ 0 & 0 & 0.1580 \end{bmatrix}$$

$$[\Phi] = \begin{bmatrix} 0.1882 & 0.9508 & 0.1507 \\ -0.6247 & -0.0337 & 0.6705 \\ 0.7579 & -0.3079 & 0.7264 \end{bmatrix} \tag{18}$$

If one were to use no knowledge of the original model to determine the mass and stiffness of the new system, an infinte number of satisfactory matrices could be found. However, all of these matrices are scaled versions of the same two matrices. Each set of the following matrices yields the modal matrices given earlier for the damaged system:

$$[M] = \frac{1}{d^2}\frac{1}{6}\begin{bmatrix} 12 & 2 & 0 \\ 2 & 12 & 4 \\ 0 & 4 & 8 \end{bmatrix}$$

$$[K] = \frac{1}{d^2}\begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & -4 \\ 0 & -4 & 4 \end{bmatrix} \tag{19}$$

where d is an arbitrary scaling factor to the system matrices. It is now necessary to determine which of the infinite number of possibilities are the actual matrices to the damaged system. To do this, the updating problem can be constrained such that the first element is not damaged. By substituting in the undamaged mass and stiffness values for the first element, the damaged system must follow the form of

$$[M] = \frac{1}{6}\begin{bmatrix} 2+2m_2 & m_2 & 0 \\ m_2 & 2m_2+2m_3 & m_3 \\ 0 & m_3 & 2m_3 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 1+EA_2 & -EA_2 & 0 \\ -EA_2 & EA_2+EA_3 & -EA_3 \\ 0 & -EA_3 & EA_3 \end{bmatrix} \tag{20}$$

We can now combine Eqs. (20) and (19) to determine the new values of m_2, EA_2, m_3, and EA_3:

$$[M] = \frac{1}{d^2}\frac{1}{6}\begin{bmatrix} 12 & 2 & 0 \\ 2 & 12 & 4 \\ 0 & 4 & 8 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2+2m_2 & m_2 & 0 \\ m_2 & 2m_2+2m_3 & m_3 \\ 0 & m_3 & 2m_3 \end{bmatrix} \tag{21}$$

$$[K] = \frac{1}{d^2}\begin{bmatrix} 5 & -1 & 0 \\ -1 & 5 & -4 \\ 0 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 1+EA_2 & -EA_2 & 0 \\ -EA_2 & EA_2+EA_3 & -EA_3 \\ 0 & -EA_3 & EA_3 \end{bmatrix} \tag{22}$$

Solving for the unknown quantities gives $m_2 = 0.5$, $EA_2 = 0.25$, and $m_3 = EA_3 = 1$, which is exactly the damage that was originally entered into the system.

It is also possible to constrain the second element from being damaged. In this case, Eq. (20) becomes

$$[M] = \frac{1}{6}\begin{bmatrix} 2m_1+2 & 1 & 0 \\ 1 & 2+2m_3 & m_3 \\ 0 & m_3 & 2m_3 \end{bmatrix}$$

$$[K] = \begin{bmatrix} EA_1+1 & -1 & 0 \\ -1 & 1+EA_3 & -EA_3 \\ 0 & -EA_3 & EA_3 \end{bmatrix} \tag{23}$$

Solving these equations for the unknown mass and stiffness values gives $m_1 = m_3 = 2$ and $EA_1 = EA_3 = 4$. It can be seen that the updating process gives a uniform increase in both the mass and stiffness of elements 1 and 3. Although it is possible that both elements are equally damaged, it is more likely that the constrained element (element 2) is really the damaged element. The uniform change in mass and stiffness is even more apparent for larger models where the number of elements is increased. To obtain the correct updated values, the model updating algorithm can be rerun with the correct element being constrained.

Conclusions

It has been shown that, although it is impossible to simultaneously update both the full mass and stiffness matrices, it is possible to update both matrices simultaneously if a constraint is added to the problem. Furthermore, it has been shown that this constraint does not drastically interfere with the capability of detecting damage in the mass and stiffness matrices.

References

[1] Baruch, M., "Optimal Correction of Mass and Stiffness Matrices Using Measured Modes," *AIAA Journal*, Vol. 20, No. 11, 1982, pp. 1623–1626.

[2] Kiddy, J., and Pines, D. J., "Damage Detection of Main Rotor Faults Using a Sensitivity Based Approach," *Proceedings of SPIE Smart Structures and Materials 1997*, Vol. 3041, 1997, pp. 611–618.

[3] Baruch, M., "Modal Data Are Insufficient for Identification of Both Mass and Stiffness Matrices," *AIAA Journal*, Vol. 35, No. 11, 1997, pp. 1797, 1798.

A. Berman
Associate Editor